Models of Flavor with Discrete Symmetries

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Abstract

In an attempt to understand the observed patterns of lepton and quark masses, models invoking a flavor symmetry G_f, under which the Standard Model generations are charged, have been proposed. One particularly successful symmetry, U(2), has been extensively discussed in the literature. The Yukawa matrices in models based on this symmetry reproduce the observed mass ratios in the lepton and quark sectors. The features of the symmetry that determine the texture of the Yukawa matrices can be found in other symmetries as well. We present a model based on a minimal, non-Abelian discrete symmetry that reproduces the Yukawa matrices associated with U(2) theories of flavor. In addition to reproducing the mass and mixing angle relations obtained in such theories, the different representation structure of our new horizontal symmetry allows for solutions to the solar and atmospheric neutrino problems.

Introduction In this talk we discuss the possibility of using discrete symmetries to construct models of flavor. We start with the observation that a U(2) symmetry has been used [1] to construct a successful model of quarks and charged leptons where all the mass ratios and mixing angles are generated via two small parameters associated with the breaking of the flavor symmetry. It is interesting to ask whether there is a smaller symmetry that can reproduce the results of the U(2) model and can also be extended to incorporate the recent results on neutrino mixing. This symmetry does indeed exist and it was discussed in [2]. There it was shown that using $T' \times Z_3$ symmetry one can construct a viable and minimal model of flavor. In the next section we review the basic features of the U(2) model and point out the key ingredients that a symmetry must have to generate the desired Yukawa textures. We then present the necessary steps to construct a model using the new local discrete symmetry and a minimal model is outlined. Finally some comments on possible implementations of T' beyond the minimal model are presented before concluding.

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U(2) Model The flavor symmetry group is G_f =U(2). Quarks and leptons of the first two generations are assigned to a **2** representation while the third generation fields are singlets. The assignment of the third generation as a singlet is motivated by the heaviness of the top quark. Putting the first two generations in a doublet yields degenerate scalar masses and thus the model is safe from FCNC contributions. The model contains three flavon fields: ϕ transforming as a doublet, A a singlet, and S a triplet. When these flavons acquire vevs they break the flavor symmetry. The breaking occurs in two steps, the first one is generated by the vevs of ϕ and S. This happens in such a way that a U(1) symmetry that rotates first generation fields by a phase is left unbroken. This remaining U(1) is then broken down to nothing at a somewhat lower scale by the vev of A. The result is a set of Yukawa textures described by two parameters, ϵ which is related to the vevs of ϕ and S, and ϵ' related to the vev of A (and therefore $\epsilon' < \epsilon$). For a detailed description of the model see Refs. [1] The ingredients that are key in obtaining the U(2) Yukawa textures are: the **1**, **2**, and **3** representations of U(2) are used in the model; the multiplication rule $\mathbf{2} \otimes \mathbf{2}$ puts the vevs of A in the right place and with the right sign; the existence of a U(1) subgroup that rotates first generation fields by a phase. These are the key ingredients that a smaller symmetry must contain in order to reproduce the successful textures of the U(2) model.

T' The group T' is the smallest with **1**, **2**, and **3** dimensional representations with the desired multiplication rule. [2] Therefore we use T' to construct the minimal model. The representations are $\mathbf{1}^0$, $\mathbf{1}^{\pm}$,$\mathbf{2}^0$, $\mathbf{2}^{\pm}$, and **3**. The superscripts add modulo 3. The remaining step is to determine whether or not it is possible to find a subgroup that allows one to break the symmetry sequentially and generate the desired textures. T' has a Z_3 subgroup which can be used as the remaining symmetry during the first breaking, namely, a symmetry that rotates first generation fields by a phase. The two-dimensional representation matrix of the element that generates this subgroup and that corresponds to the desired rotation turns out to correspond to $\mathbf{2}^-$. Unlike the U(2) model however, there is an additional condition that must be satisfied which did not exist before. We argued that it would be interesting to find a smaller symmetry, hence a discrete symmetry is desirable. Furthermore, we are interested in the possibility of having a "local" discrete symmetry. This is motivated by several arguments that global symmetries are violated by quantum gravitational effects. [3] If this is the case we need to make sure the model is anomaly free. This can be done by noting that T' is a subgroup of SU(2) and thus can be embedded in it. If we do this, then the only constraint on the model is that the matter fields fill out complete SU(2) representations, which correspond to the $\mathbf{2}^0$ and $\mathbf{1}^0$ reps of T' (see [2] for details). This, together with the fact that the desired subgroup must rotate first generations fields by a phase, leads us to extend the flavor symmetry to $G_f = T' \times Z_3$, this is the smallest group that has the desired features. The two step breaking now can take place, where the middle step symmetry is the diagonal Z_3^D subgroup of G_f. We assume that the Z_3 factor may be embedded in a U(1) gauge symmetry whose anomalies are canceled by the Green-Schwarz mechanism [4].

A Model The three generations of matter fields are assigned to the representations $\mathbf{2}0- \oplus \mathbf{1}^{00}$ (the second triality corresponds to the Z_3 and also adds modulo 3). The Higgs fields $H_{U,D}$ transform as singlets. The Yukawa mass matrices can now be obtained and we introduce three flavons A, ϕ, and S with the representations $\mathbf{1}^{0-}$, $\mathbf{2}^{0+}$, and $\mathbf{3}^{-}$ respectively. Again, the vevs of S and ϕ are assumed to break $T' \times Z_3$ down to Z_3^D putting entries of O(ϵ) in the Yukawa matrices, and then finally the vev of A breaks the remaining Z_3^D down to nothing yielding entries of O(ϵ'. These considerations yield the textures

$$Y_{U,D,L} \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} , \tag{1}$$

where O(1) coefficients have been omitted. These are the same textures of the U(2) model, as desired. As a note we mention that in order to differentiate between the up-type and down-type quarks it is possible to embed both the U(2) model and the $T' \times Z_3$ model into a GUT, for example an SU(5) [2]. Now the flavons may have non-trivial transformation properties under the GUT symmetry and the textures are accordingly modified. From now on the discussion will concentrate on this "GUT-model" version. Now that we have reproduced the U(2) model, neutrinos are introduced into the model. Three generations of right-handed neutrinos are introduced with the assignment $\mathbf{2}^{0-} \oplus \mathbf{1}^{-+}$. This assignment leads to Dirac and Majorana mass matrices that allow the introduction of flavons that do not contribute at all to the charged fermion mass matrices. Two such flavons are introduced transforming as $\mathbf{2}^{+0}$ and yielding the following Dirac and Majorana mass matrices:

$$\begin{aligned} M_{LR} &\approx \begin{pmatrix} 0 & l_1\epsilon' & l_5 r_2 \epsilon' \\ -l_1\epsilon' & l_2\epsilon^2 & l_3 r_1 \epsilon \\ 0 & l_4\epsilon & 0 \end{pmatrix} \langle H_U \rangle , \\ M_{RR} &\approx \begin{pmatrix} r_3 {\epsilon'}^2 & r_4 \epsilon\epsilon' & r_2\epsilon' \\ r_4\epsilon\epsilon' & r_5\epsilon^2 & r_1\epsilon \\ r_2\epsilon' & r_1\epsilon & 0 \end{pmatrix} \Lambda_R , \end{aligned} \tag{2}$$

where O(1) coefficients have been introduced and Λ_R is the right-handed neutrino scale. Using the seesaw mechanism one obtains the texture

$$M_{LL} \sim \begin{pmatrix} (\epsilon'/\epsilon)^2 & \epsilon'/\epsilon & \epsilon'/\epsilon \\ \epsilon'/\epsilon & 1 & 1 \\ \epsilon'/\epsilon & 1 & 1 \end{pmatrix} \frac{\langle H_U \rangle^2}{\Lambda_R} . \tag{3}$$

This texture leads naturally to large mixing between second and third generation neutrinos. The $1-2$ mixing is of $\mathrm{O}(\epsilon'/\epsilon)$, which can be accommodated to give the bimaximal solution with the use of the O(1) coefficients (in order to determine the mixings accurately one computes the CKM matrix for the lepton sector). In [2] we presented a detailed numerical analysis of this model consisting of a fit to the experimental data. This fit contained a renormalization group analysis and a ξ^2 minimization in order to prove that a set of O(1) coefficients could be found that reproduced the experimental data.

Alternative uses Here we comment on the possibility of using the group T' in different models of flavor. In particular, it can be used as a global symmetry. In this case there is no need for an extra Z_3 and it is possible to have a model that reproduces the U(2) model and accommodates the solutions to the atmospheric and solar neutrino deficits [2]. This is an interesting result when one notes that T' is a subgroup of SU(2), which is known not to lead to a good theory of flavor unless flavor universality is assumed. Another example in which T' can be used is to consider the model based on the local $T' \times Z_6$ symmetry presented in [2]. In this model it is not necessary to have a GUT in order to explain the differences between the up- and down-type quark sectors of the theory. Furthermore, this model also predicts the ratio of m_t/m_b, which in the models described above it is put in by hand. This model also accommodates the neutrino results.

Conclusion Models based on T' flavor symmetry were discussed. In particular a minimal model with $G_f = T' \times Z_3$ that reproduces the U(2) textures for fermion masses was reviewed. This model can also accommodate the results on neutrino oscillations. The main ingredients in the construction of the model were discussed.

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